Solution to Quiz 4

MATH 1231 – Single-variable Calculus I

Summer 2016

1. Circle the correct answer (2 points)

   (a) Let g is defined by

   $$g(x) = \int_{x^3}^{\sqrt{x}} \sin(t)\ dt$$

   Then, the derivative of g, $g'(x) = \sin(\sqrt{x}).\frac{1}{2\sqrt{x}} - \sin(x^3).3x^2$

   (b) $\int_3^5 (x^3 - 3\sin x)dx =$

   $$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left[\left(3 + \frac{2i}{n}\right)^3 - 3\sin\left(3 + \frac{2i}{n}\right) \right]$$

2. Find the following limit (4 points)

   $$\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2$$

   Solution: Method 1: Observe that the given expression is the limit of the Riemann Sum of the function x^2 on the interval $[0, 2]$. So, the answer is

   $$\int_0^2 x^2\ dx = \frac{x^3}{3}\Big]_0^2 = \frac{8}{3} - 0 = \frac{8}{3}$$

   Method 2: Simplify the expression

   $$\begin{aligned}\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{2i}{n}\right)^2 &= \lim_{n\to\infty} \frac{8}{n^3}.\sum_{i=1}^{n} i^2 \\ &= \lim_{n\to\infty} \frac{8}{n^3} \frac{n(n+1)(2n+1)}{6} \\ &= \frac{8}{6} \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{n^3} \\ &= \frac{8}{6}.2 \\ &= \frac{8}{3}\end{aligned}$$

3. Find the following indefinite integral with the substitution $y = 1 + \cos(t)$ (4 poitns)

   $$\int \sin t\ \sqrt{1 + \cos t}\ dt$$

Solution: Substitution

$$y = 1 + \cos t$$
$$\frac{dy}{dt} = -\sin t$$
$$dy = -\sin t \ dt$$

Now,

$$\begin{aligned}
\int \sin t\sqrt{1+\cos t} \ dt &= \int \sqrt{y} \sin t \ dt \\
&= -\int \sqrt{y} \ dy \\
&= -\frac{y^{\frac{3}{2}}}{\frac{3}{2}} + C \\
&= -\frac{2}{3}(1 + cost)^{\frac{3}{2}} + C
\end{aligned}$$

where C is a constant