Solution to Quiz 1

MATH 1231 – Single-variable Calculus I

Summer 2016

1. Determine whether the following function is odd, even or neither **(4 points)**

$$f(x) = x^2|x^3|$$

Solution: A function f is odd if $f(-x) = -f(x)$ and even if $f(-x) = f(x)$ for all x. So we calculate $f(-x)$.

$$\begin{aligned} f(-x) =& (-x)^2|(-x)^3| \\ =& x^2|-x^3| \\ =& x^2|x^3| \\ =& f(x) \end{aligned}$$

Hence, the given function is even.

2. Let a function be given by **(6 points)**

$$f(t) = \frac{t^2 - 9}{t^2 + 2t - 15}$$

(a) Find the domain of f. (2 points)

Solution: The domain is the set on which the function is defined. The function is not defined for those t for which the denominator is zero, that is,

$$\begin{aligned} t^2 + 2t - 15 =& 0 \\ \implies t^2 + 5t - 3t - 15 =& 0 \\ \implies (t+5)(t-3) =& 0 \\ \implies t =& (-5), 3 \end{aligned}$$

So, the domain of the function f is $\mathbb{R} \setminus \{-5, 3\}$.
Or, in the interval notation, the domain is $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$.

(b) Find the following limit (2 points)

$$\lim_{t \to 3} f(t)$$

Solution:

$$\begin{aligned} \lim_{t \to 3} f(t) =& \lim_{t \to 3} \frac{t^2 - 9}{t^2 + 2t - 15} \\ =& \lim_{t \to 3} \frac{(t+3)(t-3)}{(t+5)(t-3)} \\ =& \lim_{t \to 3} \frac{(t+3)}{(t+5)} \\ =& \frac{3+3}{3+5} \\ =& \frac{6}{8} \\ =& \frac{3}{4} \end{aligned}$$

(c) Let a function g be defined by (2 points)

$$g(t) = \begin{cases} \frac{t^2-9}{t^2+2t-15} & \text{if } t \in [0,3) \cup (3,\infty) \\ \\ \frac{3}{4} & \text{if } t = 3 \end{cases}$$

Is g continuous at $t = 3$? Justify your answer.

Solution:

$$\begin{aligned} \lim_{t \to 3} g(t) &= \lim_{t \to 3} \frac{t^2 - 9}{t^2 + 2t - 15} && \text{(since } t \to 3 \text{ i.e. } t \neq 3\text{, we choose the first definition in } g\text{)} \\ &= \frac{3}{4} && \text{(calculated in part (b))} \end{aligned}$$

By the definition of g,

$$g(3) = \frac{3}{4}$$

Hence, we have,

i. $\lim_{t \to 3} g(t)$ exists.
ii. $\lim_{t \to 3} g(t) = g(3)$.

So, g is continuous at $t = 3$.