Midterm

MATH 1231 – Single-variable Calculus I

Summer 2016

Total Points: 110 **Total Time:** 90 minutes
Max Points: 100

Name: _____ **Date:** _____

Read all of the following information before starting the exam:

- There are **11 questions** and each question carries 10 points. The maximum points that you can get is **100**.
- Show all work, clearly and in order, to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Do not use a calculator.
- Books, notes or any other help are not allowed.
- Do all your scratch work on the last sheet of this booklet.
- Circle or otherwise indicate your final answers.

Signature: _____

(By signing above you agree to abide by the Code of Academic Integrity.)

1. Find the domain of the function

$$f(x) = \frac{x}{\sqrt[3]{x^2 + x - 6}}$$

2. Find the limit

$$\lim_{x \to 1} \frac{1 - \sqrt{x}}{x - x^2}$$

3. (a) Let g be a differentiable function with $g'(x) \geq 7$ for all x. If $g(3) = 2$ how big can $g(1)$ be?
(Hint: Use the Mean Value Theorem)

(b) Determine whether the following function is odd, even, or neither.

$$f(x) = (x^2 + 3)(x^3 - 5x)$$

4. Find the limit

$$\lim_{x \to 0} x^4 \cos\left(\frac{\pi}{x^3}\right)$$

(Hint: Use the Sandwich Theorem)

5. Differentiate the function

$$f(x) = x \sin\left(\frac{3}{x}\right)$$

6. Differentiate the function

$$g(y) = \frac{y^2}{y+4}$$

7. Find the equation of the tangent line through $(0, 0)$ to the graph of the equation

$$y = \sin(5x) + \sin^2(5x)$$

8. Find $\frac{dy}{dx}$ by implicit differentiation

$$\cos(xy) = 2xy^2 + \sin(y)$$

9. Let f be a function given by

$$f(x) = x^3 + \frac{x^2}{2} + 2x + 7$$

(a) Show that f has a root in the interval $[-2, 2]$.
(Hint: Use the Intermediate Value Theorem)

(b) Show that f does not have more than one root in the interval $[-2, 2]$.
(Hint: Use Rolle's Theorem)

10. Let f be a function given by

$$f(x) = \frac{x^3}{3} - x + 3$$

(a) Find the critical points of f.

(b) Find the intervals where f is increasing and where f is decreasing. Determine whether the critical points are local max, local min or neither.

11. Find all values of c such that the function

$$f(x) = \begin{cases} c^2x^2 + 2x + 1 & \text{if } x \leq 2 \\ 2xc + 11 & \text{if } x > 2 \end{cases}$$

is continuous everywhere.

Scratch

Scratch