

Final Examination
MATH 1231 – Single-variable Calculus I
Summer 2016

Total Points: 110
Max Points: 100

Total Time: 90 minutes

Name: _____

Date: _____

Read all of the following information before starting the exam:

- There are **10 questions** in this booklet and the total points are **110**. The maximum points that you can score is **100**.
- The first question carries 20 points the rest carry 10 points each.
- Show all work, clearly and in order, to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Do not use a calculator or any other electronic device.
- Books, notes or any other help are not allowed.
- Do all your scratch work on the last sheet of this booklet.
- Circle or otherwise indicate your final answers.

Signature: _____

(By signing above you agree to abide by the Code of Academic Integrity.)

1. Let

$$f(x) = \frac{x^3}{x - 2}$$

(a) Find a slant asymptote of f .

(5 points)

(b) Differentiate the function f .

(5 points)

(c) Find the interval(s) on which f is increasing and the interval(s) on which f is decreasing.
(5 points)

(d) Find the critical point(s) of f and determine whether each critical point is a local maximum, local minimum or neither.
(5 points)

2. Find the equation of the tangent line to the graph

(10 points)

$$y = x^3 - 7x + 1$$

at the point $(0, 1)$.

3. Let $f''(x) = 3x^2 - \sin x$, $f(0) = \frac{\pi^4}{64}$, $f(\frac{\pi}{2}) = 0$. Find $f(x)$. (10 points)

4. Let a function f be given by

(10 points)

$$f = \begin{cases} 3x - 2 & \text{if } x < 1 \\ 1 & \text{if } x = 1 \\ x^2 & \text{if } x > 1 \end{cases}$$

Determine whether f is continuous everywhere or not. Justify your answer.

5. Find the following limit

(10 points)

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{3}{n} \left(\frac{3i}{n} \right)^3$$

6. Let

(10 points)

$$g(x) = \int_{2x}^{\sin x^2} \sqrt{1 + \cos t} \, dt$$

Find the derivative of g , $g'(x)$.

7. Evaluate the following definite integral

(10 points)

$$\int_0^{\frac{\pi}{4}} \frac{\sin^2 x \cdot \cos x + \cos^3 x}{\cos^3 x} dx$$

8. (a) Find the general indefinite integral

(5 points)

$$\int \sin x (2 + \cos x)^{\frac{3}{2}} dx$$

(b) Evaluate the following indefinite integral

(5 points)

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} 3 \sin(3x) dx$$

9. Suzie wants to build a rectangular backyard fenced on 3 sides using 120 feet of fence. What dimension (length and width) of the backyard will maximize its area? (10 points)

10. (a) Find the point(s) where the graphs of the following functions intersect. (5 points)

$$y = x^2$$

$$y = x^3$$

- (b) Compute the area of the bounded region enclosed by the graphs of the given functions. (5 points)

Scratch

Scratch