Course #
MAP-499
Professor Bhattacharya

Entanglement of non-convex granular media

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Introduction

Granular Media is a material composed of discrete solid particles, with solid-like or fluid-like behavior depending on their arrangement.

Granular entanglement describes the jamming behavior that emerges from the overlap between convex hulls of non-convex particles. Our work employs computational modeling and bulk simulations to quantify and compare the entanglement capacity of various parametrized family of non-convex geometries.

We identify features of particle geometry such as **accessible volume** and **angular coverage** that contribute to stronger entanglement. Varying degrees of entanglement control structural stability, offering shape-driven cohesion for reconfigurable engineering structures without adhesives.

Conclusions

Our current result confirm that **granular accessibility** is a metric closely related to changes in geometry and reflect the degree of openness of the structures.

Considering **settling height** as a measure of entanglement, our simulations suggest that accessibility may be able to predict interlocking behavior. Further work will focus on 3D equivalent of similar problems.

Acknowledgments & References

This project is developed under the mentorship of Professor Debdeep Bhattacharya, who proposed the topic, provided foundational code and guidance. We also acknowledge the use of computational resources provided by Grinnell College's High Performance Computing cluster.

1) Bhattacharya, D., & Lipton, R. P. (2023). Simulating Grain Shape Effects and Damage in Granular Media Using PeriDEM. *SIAM Journal on Scientific Computing*, *45*(1), B1–B26.
2) Gravish, N., Franklin, S. V., Hu, D. L., & Goldman, D. I. (2012). Entangled Granular Media. *Physical Review Letters*, *108*(20).

Methods

Mathematical concept: differential equations

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t) = \boldsymbol{F}_i^{int}(\mathbf{x},t;\mathbf{u}) + \boldsymbol{F}_i^{ext}(\mathbf{x},t;\mathbf{u})\ \forall(\mathbf{x},t)$$

Where $\rho(x)$ is the material density at position x, $\ddot{u}(x,t)$ is the acceleration, $F^{int}(\text{x, t})$ is the internal peridynamic force, and $F^{ext}(\text{x, t})$ is the external Discrete-Element force between every two particles.

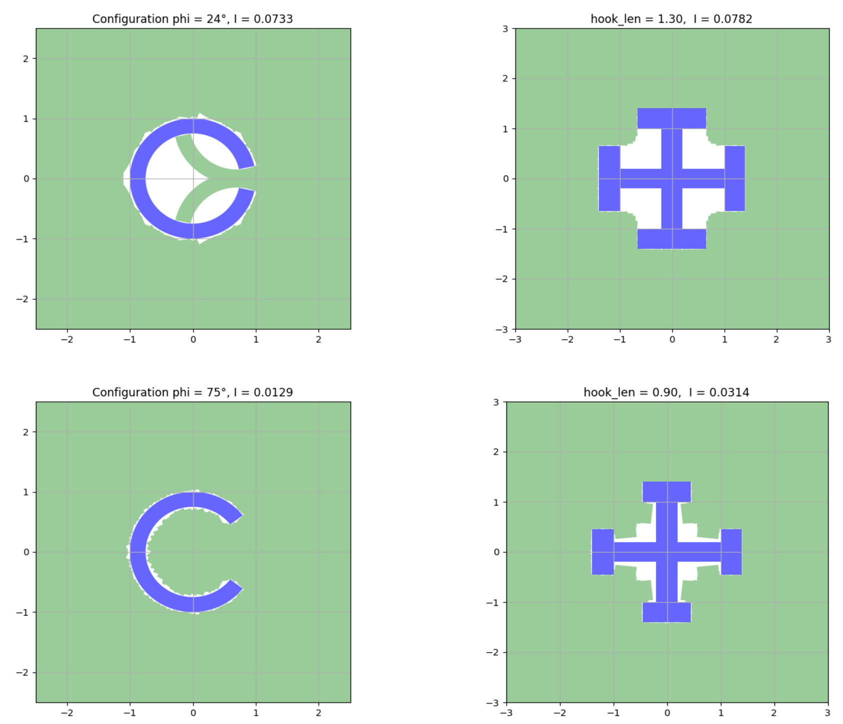
Definitions:

- Accessible area

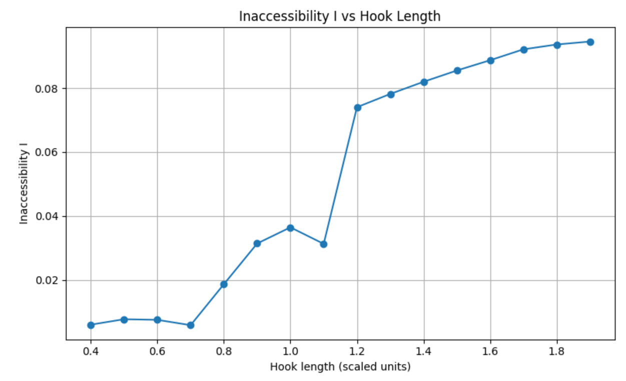
the area which could be reached by the other object.

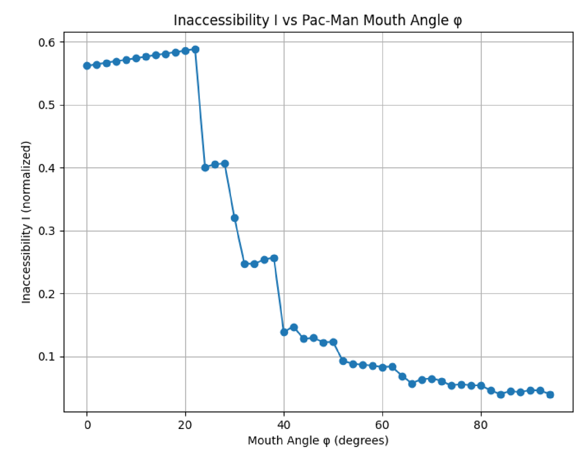
- Inaccessibility

the ratio of inaccessible area to the whole area. We hypothesized that the greater the inaccessibility, less possible one object is to be entangled with another.



Numerically computed accessible regions for several particle geometries





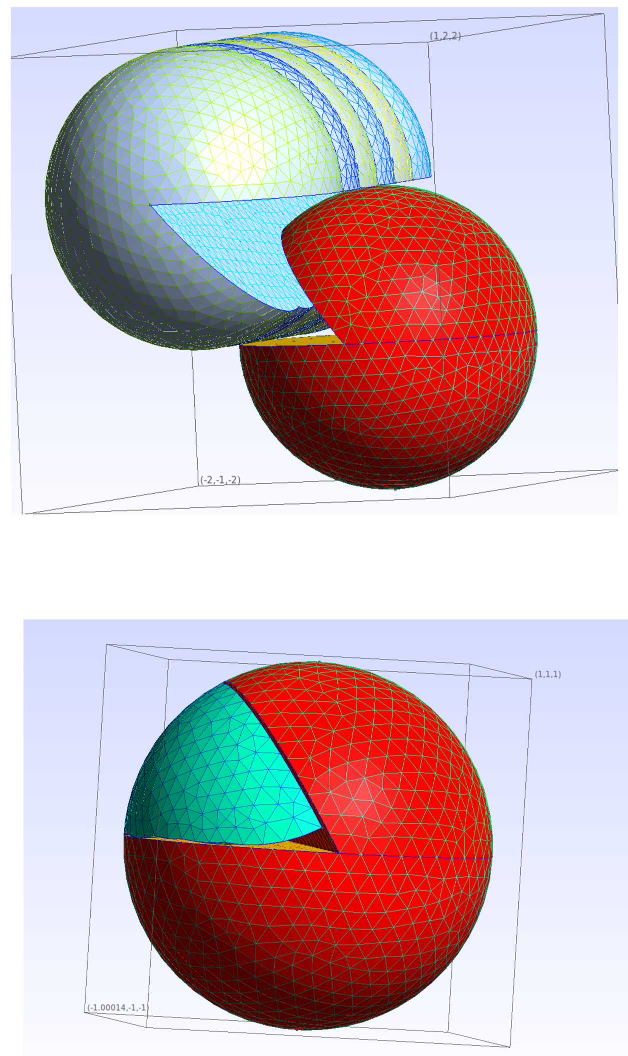
Accessibility vs opening angle for C and Pacman

Simulation:

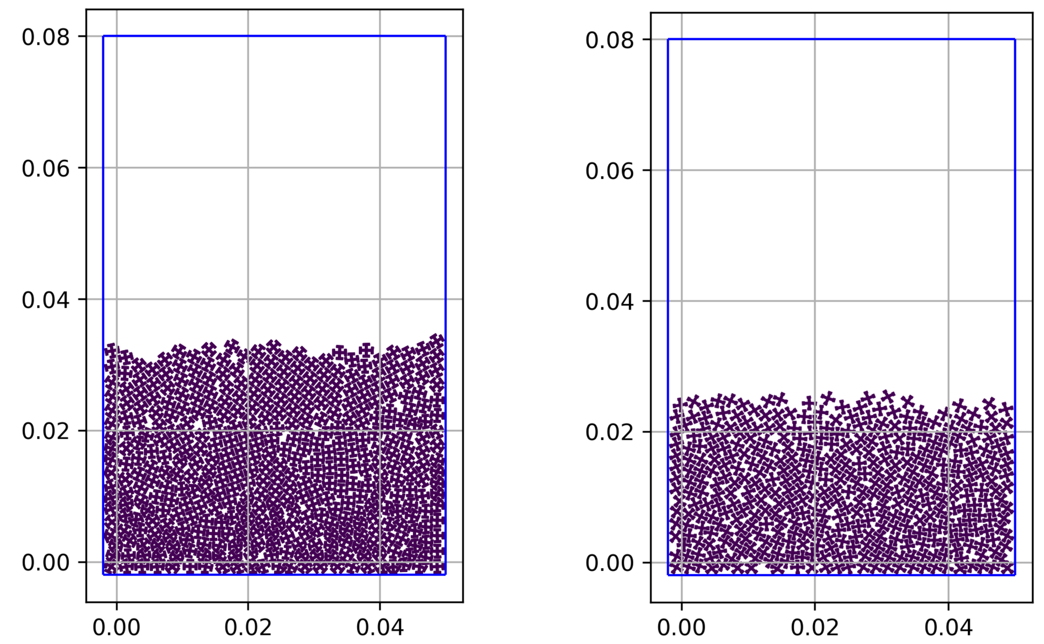
We set up a bulk simulation in a container with 400 non-convex particles. After allowing the particles to fall due to gravity and to settle down, a force is applied to the container to have the particles realigned. By measuring the height difference before and after the settling, we plot the height difference against the accessibility.

We defined the degree of entanglement as γ, which is (1- φ) where φ is the bulk volume fraction: φ = α/*i*.

The settled height α is the final height the bulk achieves, while the unsettled height *i* is the original height of the bulk, the same as the height of the container.



Accessible volume for 3D Pacman shape against all rigid motions of an interacting particle



Snapshot of a settling simulation of nonconvex particles under gravity
Left figure: less accessibility
Right figure: more accessibility
(40 cores, ~30 mins per simulation)

Results

- We developed **two algorithms** for generating one-parameter families of shapes using python (pygmsh and shapely) and gmsh:
  - By prescribing a polygon to directly define a shape by specifying a sequence of points
  - Using volume primitives such as addSphere, addBox, and addCylinder to construct basic geometries, apply Boolean operations like cut, fuse, and intersect to modify them, and perform union using fuse to merge multiple volumes before intersection
- We developed a sweeping algorithm to compute accessible area/volume in 2D and 3D grains.
- **Opening angle vs accessible area/volume:** the shapes with higher opening angle results in higher accessibility.
- **3D-printed shapes for demonstration and visualization:** We generated a dataset to tackle an inverse problem: designing particle to have prescribed bulk entanglement: non-convex shapes with higher accessibility results in stronger entanglement.
- Structures with higher accessibility have a larger settling height difference, which means they are more likely to provide lower degree of entanglement.